University at Buffalo
Yorck Sommerhäuser

Spring Semester 2015
MTH 620: Sheet 9

Algebra II

Problem 1: Suppose that F is a field of characteristic $p > 0$ and that a is not in the image of the Frobenius homomorphism, i.e., $a \in F \setminus F^p$. Show that the polynomial $x^p - a \in F[x]$ is irreducible. (4 points)

Problem 2: Suppose that F is a field of characteristic $p > 0$. Show that the following conditions on F are equivalent:

1. Every algebraic extension of F is separable.
2. The Frobenius homomorphism is surjective.

Fields that satisfy these equivalent conditions are called perfect. (6 points)
(Hint: Write minimum polynomials in the form $f\left(x^{p^e}\right)$, where e is as large as possible. Under which conditions on e is the polynomial separable?)

Problem 3: Suppose that F is a field of characteristic $p > 0$. Show that the field extension $F(x^p) \subset F(x)$ is purely inseparable. (6 points)
(Hint: Use Problem 3 on Sheet 8. Show first that the extension is inseparable.)

Problem 4: Suppose that F is a field with seven elements and that E is the splitting field of the polynomial $f(x) = x^6 + 3 \in F[x]$. Find the degree $[E : F]$. (From the Bavarian State Exam for High School Teachers in Spring 1994). (4 points)

Due date: Wednesday, April 29, 2015. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.