University at Buffalo
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Spring Semester 2015
MTH 620: Sheet 5

Algebra II

Problem 1: For a field K, consider the ring $K[x^2, x^3]$, i.e., the image of the substitution homomorphism

$$K[x, y] \to K[x], x \mapsto x^2, y \mapsto x^3$$

1. Show that this ring consists exactly of those polynomials whose coefficient of x vanishes. (1 point)
2. Show that this ring is not a unique factorization domain. (3 points)

Problem 2: Find the greatest common divisor of the polynomials

$$p(x) := x^5 + 2x^3 + x^2 + x + 1 \quad \text{and} \quad q(x) := x^5 + x^4 + 2x^3 + 2x^2 + 2x + 1$$

in $\mathbb{Q}[x]$. (4 points)

Problem 3:

1. Decide whether or not the polynomial $f(x) := x^4 + 10x + 5$ is irreducible in $\mathbb{Z}[x]$. If it is not irreducible, decompose it into irreducible factors. (1 point)
2. Decide whether or not the polynomial $g(x, y) := x^2 + y^2 - 1$ is irreducible in $\mathbb{Q}[x, y]$. If it is not irreducible, decompose it into irreducible factors. (2 points)

Problem 4:

1. Decide whether or not the polynomial $p(x) := x^4 + 4x^3 + 6x^2 + 2x + 1$ is irreducible in $\mathbb{Z}[x]$. If it is not irreducible, decompose it into irreducible factors. (3 points)
2. Decide whether or not the polynomial $q(x) := x^5 + x^4 + x^2 + x + 2$ is irreducible in $\mathbb{Z}[x]$. If it is not irreducible, decompose it into irreducible factors. (6 points)

(Hint: Problem 4 is not easy. Kronecker's method, described in the exercises in Hungerford's book, can be helpful for the second polynomial.)

Due date: Wednesday, March 11, 2015. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.