University at Buffalo **Fall Semester 2014**
Yorck Sommerhäuser **MTH 619: Sheet 5**

Algebra I

Problem 1:

1. Show that a cyclic p-group is indecomposable (cf. Jacobson, p. 152/153). (1 point)
2. Explain how the Krull-Remak-Schmidt theorem (cf. Jacobson, p. 156) can be used to show that the groups
$$C_{15} \times C_5 \times C_5 \times C_5 \times C_{25} \times C_{25}$$
and
$$C_3 \times C_5 \times C_5 \times C_{25} \times C_{25} \times C_{25}$$
are not isomorphic. (2 points)

Problem 2: List all abelian groups of order 8281, up to isomorphism. (I.e., make a list of abelian groups of order 8281 so that no two groups on this list are isomorphic and every abelian group of order 8281 is isomorphic to a group on this list.) For each group, list the elementary divisors and the invariant factors. (4 points)

Problem 3: Let p be a fixed prime, and suppose that
$$G = A_1 \times A_2 \times \ldots \times A_n$$
is a direct product of cyclic p-groups. Show that G cannot be generated by fewer than n elements. (5 points)

Problem 4:

1. Write down all elements of $\mathrm{Aut}(C_{24})$ explicitly by stating what the image of a generator of C_{24} under every automorphism is. (1 point)
2. Show that $\mathrm{Aut}(C_{24})$ is an elementary abelian 2-group. (2 points)
3. Find all natural numbers n for which $\mathrm{Aut}(C_n)$ is an elementary abelian 2-group. (5 points)

Due date: Monday, October 13, 2014. Please work in teams of two students. Every submitted solution should carry exactly two names, and each team member should have written up two of the problems. Please write your solution on letter-sized paper. It is not necessary to submit this sheet with your solution.