University at Buffalo **Fall Semester 2014**
Yorck Sommerhäuser **MTH 619: Sheet 3**

Algebra I

For a group G, the higher commutator groups are defined as $G^1 := G'$, $G^2 := G''$, $G^3 := G'''$, and inductively $G^{n+1} := (G^n)'$. A group is called solvable if some higher commutator group is equal to $\{1\}$, i.e., if there exists $n \in \mathbb{N}$ such that $G^n = \{1\}$ (cf. Thm. 6.1.5, p. 123). G is called metabelian if $G'' = \{1\}$.

Problem 1: Suppose that $G = S_3$, the set of bijective mappings from the set $\{1, 2, 3\}$ to itself.

1. List explicitly the elements of the commutator group G'. (2 points)
2. List explicitly the elements of the second commutator group G'' and also of all higher commutator groups. (2 points)
3. Decide whether G is solvable or metabelian. (1 point)

Problem 2: Suppose that $G = D_{2n}$, the dihedral group considered in Problem 1 on Sheet 1.

1. List explicitly the elements of the commutator group G'. (2 points)
2. List explicitly the elements of the second commutator group G'' and also of all higher commutator groups. (2 points)
3. Decide whether G is solvable or metabelian. (1 point)

Problem 3: For an arbitrary group G and a cyclic group C_n, find a (very natural) bijection between $\mathrm{Hom}(G, C_n)$ and $\mathrm{Hom}(G/G', C_n)$. (The group G/G' is called the commutator factor group, or commutator quotient group.) (4 points)

Problem 4:

1. Show that $C_n \times C_m$ is cyclic if m and n are relatively prime. (3 points)
2. Show that $C_n \times C_m$ is not cyclic if m and n are not relatively prime. (3 points)

Due date: Monday, September 22, 2014. Please work in teams of two students. Every submitted solution should carry exactly two names, and each team member should have written up two of the problems. Please write your solution on letter-sized paper. It is not necessary to submit this sheet with your solution.