Algebra I

Problem 1: Suppose that G is a group in which every element that is different from the unit element has order 2. Show that G is abelian.   (3 points)

Problem 2: Suppose that G is an abelian group of order p^2, where p is a prime. Show that $G \cong C_{p^2}$ or $G \cong C_p \times C_p$.   (7 points)

Problem 3: Suppose that G is a group of order 4. Show that $G \cong C_4$ or $G \cong C_2 \times C_2$.   (4 points)

Problem 4: Find all subgroups of the quaternion group Q_8 and decide which of them are normal.   (6 points)

Due date: Monday, September 15, 2014. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.