Memorial University of Newfoundland **Winter Semester 2017**
Yorck Sommerhäuser **MATH 3300: Sheet 5**

Set Theory

Problem 1: Let m and n be natural numbers; i.e., elements of the set ω defined in class.

1. If $m < n$, show that $m^+ \leq n$. (15 points)
2. If $m < n$, show that $m^+ < n^+$. (5 points)

Problem 2: Let m, n and k be natural numbers. We have seen in class that $m < n$ implies that $m + k < n + k$, and you can use that in the following problems. Here, show that also $mk < nk$ if $k \neq 0$. (20 points)

Problem 3: For natural numbers m and n, show that the following conditions are equivalent:

1. $m \leq n$
2. There exists $k \in \omega$ with $n = m + k$. (30 points)

Problem 4: Suppose that $E \subset \omega$ is a non-empty set of natural numbers. Show that there exists an element k in E such that $k \leq m$ for all $m \in E$.
(Hint: This is not easy. Consider the set

$$S := \{n \in \omega \mid \forall\, m \in E : n \leq m\}$$

and assume that $E \cap S = \emptyset$. Show that then S is a successor set and derive a contradiction.) (40 points)

Due date: Monday, March 6, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.