Memorial University of Newfoundland **Winter Semester 2017**
Yorck Sommerhäuser **MATH 3300: Sheet 4**

Set Theory

Problem 1: Suppose that $f : X \to Y$ and $g : Y \to Z$ are functions.

1. Show that $g \circ f$ is injective if f and g are both injective. (5 points)
2. Show that $g \circ f$ is surjective if f and g are both surjective. (5 points)
3. Show that f is injective if $g \circ f$ is injective. (5 points)
4. Show that g is surjective if $g \circ f$ is surjective. (5 points)

Problem 2: Suppose that $f : X \to Y$ is a function. Show that the following conditions on f are equivalent:

1. f is injective.
2. For all subsets A and B of X, we have $f(A \cap B) = f(A) \cap f(B)$. (20 points)

Problem 3: Let ω be the set of all natural numbers. If $n \neq 0$, show that there is $m \in \omega$ with $n = m^+$.
(Hint: Use induction in the form of the third Peano axiom stated on page 46 of the textbook.) (30 points)

Problem 4: Show that ω is transitive.
(Hint: Use induction as in Problem 3. The notion of a transitive set is defined on page 47 of the textbook and is slightly different from the notion of a transitive relation.) (30 points)

Due date: Monday, February 13, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.