Memorial University of Newfoundland **Winter Semester 2017**
Yorck Sommerhäuser **MATH 3300: Sheet 3**

Set Theory

Problem 1: Suppose that $f : X \to Y$ and $g : Y \to X$ are functions that satisfy $g \circ f = \mathrm{id}_X$. Show that f is injective and g is surjective. (20 points)

Problem 2: Suppose that $f : X \to Y$ is a function.

1. If $(A_i)_{i \in I}$ is a family of subsets of X, show that

$$f\left(\bigcup_{i \in I} A_i\right) = \bigcup_{i \in I} f(A_i)$$

(15 points)

2. For the sets $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4, 5, 6\}$, consider the function $f : X \to Y$ defined by

$$f(1) = 6 \qquad f(2) = 6 \qquad f(3) = 4 \qquad f(4) = 4 \qquad f(5) = 5$$

Find subsets A and B of X satisfying $f(A \cap B) \neq f(A) \cap f(B)$. (15 points)

Problem 3: Suppose that $R \subset X \times Y$ is a relation. As in Problem 2 on Sheet 2, we define the inverse relation

$$R^{-1} := \{(y, x) \in Y \times X \mid (x, y) \in R\}$$

If $S \subset Y \times Z$ is another relation, show that

$$(S \circ R)^{-1} = R^{-1} S^{-1}$$

(15 points)

Problem 4: Suppose that $R \subset X \times X$ is a relation.

1. Show that R is transitive if and only if $R \circ R \subset R$. (10 points)

2. Even if R is not necessarily transitive, define $R^n := R \circ R \circ \ldots \circ R$ (n times). Show that

$$T := \bigcup_{n=1}^{\infty} R^n$$

is equal to the transitive closure from Problem 3 on Sheet 2. (25 points)

Due date: Monday, February 6, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.