Memorial University of Newfoundland **Winter Semester 2017**
Yorck Sommerhäuser **MATH 3300: Sheet 1**

Set Theory

Problem 1: The first distributive law for sets asserts that for three sets A, B, and C, we have

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Prove this statement in the following way:

1. Find an equivalent statement that involves the assertions $x \in A$, $x \in B$, and $x \in C$.
2. Prove this equivalent statement with the help of a truth table.

(Remark: The second distributive law, which states that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

can be proved in a very similar manner.) (20 points)

Problem 2: The first of De Morgan's laws for sets asserts that for three sets A, B, and C, we have

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$$

1. Prove this statement with the same method as in Problem 1. (10 points)
2. Show in the same way that this set is also equal to $(A \setminus B) \setminus C$. (10 points)

Problem 3: The second of De Morgan's laws for sets asserts that for three sets A, B, and C, we have

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

1. Prove this statement with the same method as in Problem 1. (10 points)
2. Show in the same way that

$$A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$$

(10 points)

Problem 4: The symmetric difference of two sets A and B is defined as

$$A \bigtriangleup B = (A \setminus B) \cup (B \setminus A)$$

It satisfies $A \bigtriangleup B = B \bigtriangleup A$. Using this without proof as well as all the other rules above, including the distributive law only stated, show without using truth tables that

$$\begin{aligned}&(A \bigtriangleup B) \bigtriangleup C = \\ &(A \setminus (B \cup C)) \cup (B \setminus (A \cup C)) \cup (C \setminus (A \cup B)) \cup (A \cap B \cap C)\end{aligned}$$

To show this, you can also use without proof the rules

$$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$$

as well as $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$. (30 points)

Deduce that

$$(A \bigtriangleup B) \bigtriangleup C = A \bigtriangleup (B \bigtriangleup C)$$

(10 points)

Due date: Monday, January 23, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.

Change of syllabus: Office hours will be Monday and Friday from 1:00 pm to 3:00 pm.