Memorial University of Newfoundland **Winter Semester 2020**
Yorck Sommerhäuser **MATH 3331: Sheet 7**

Projective Geometry

Problem 1: Suppose that l is a line in a plane α, and that $f : R_l \to R_l$ is a hyperbolic projectivity with the two distinct fixed points A and B. Suppose that C is a third distinct point, and let D be the harmonic conjugate of C with respect to A and B. If $f(C) = D$, show that f is an involution. (25 points)

Problem 2: Conversely, suppose that A, B, C, D are distinct points on a line l, and that f is a projectivity from l to itself so that $ABCD \barwedge ABDC$. Show that D is the harmonic conjugate of C with respect to A and B. (25 points)

Problem 3: Suppose that l is a line in a plane α, and that A and B are points on l. Suppose that C and C' are two more points on l, distinct from A and B, and let D and D' be the harmonic conjugates of C and C' with respect to A and B. If $f = (CC')(DD')$, show that $f(A) = B$ and $f(B) = A$. (25 points)

Problem 4: Suppose that α is a plane, that $\mathcal{P}$ is the set of all points in that plane, and that $\mathcal{L}$ is the set of all lines in that plane. Suppose that $g : \mathcal{L} \to \mathcal{L}$ is a bijective map with the property that, if l, m, and n are concurrent lines, then $g(l)$, $g(m)$, and $g(n)$ are also concurrent. Show that there is a map $f : \mathcal{P} \to \mathcal{P}$ with the property that $A \in \mathcal{P}$ is incident with $l \in \mathcal{L}$, then $f(A)$ is incident with $g(l)$. (25 points)

Due date: Tuesday, March 24, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.