Memorial University of Newfoundland **Winter Semester 2020**
Yorck Sommerhäuser **MATH 3331: Sheet 6**

Projective Geometry

Problem 1: Pappus's theorem can be stated as follows: Let A, B, C be three distinct points on a line l, and let A', B', C' be three distinct points on a different line l', all of which are distinct from $l \cdot l'$. Then the three intersection points of the cross-joins

$$L := B'C \cdot BC' \qquad M := C'A \cdot CA' \qquad N := A'B \cdot AB'$$

are collinear. State the dual of this theorem and illustrate it by a picture in the plane.
(Remark: Pappus's theorem is Theorem 4.41 on page 38 in the textbook, where it is stated slightly differently.) (25 points)

Problem 2: Suppose that A, B, C are distinct points on a line l, and that D is the harmonic conjugate of C with respect to A and B. Show that there is a projectivity from l to itself so that $ABCD \barwedge ABDC$. (25 points)

Problem 3: In Figure 2.4A on page 20 of the textbook, construct a projectivity from the line l which contains the points A, B, C, D, E, F to itself with the property that $ADBF \barwedge ADCE$, and therefore fixes A and D.
(Hint: Use the point $V := PS \cdot QR$. The projectivity is the composition of two perspectivities.) (25 points)

Problem 4: Suppose that a projectivity from a line l to itself is the composition of two perspectivities. Show that there is at least one point on l that is mapped to itself.
(Hint: Of course, the perspectivities are between l and another line.) (25 points)

Due date: Tuesday, March 17, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.