Memorial University of Newfoundland **Winter Semester 2020**
Yorck Sommerhäuser **MATH 3331: Sheet 5**

Projective Geometry

Problem 1: Suppose that a and b are two distinct lines in the plane α. Show that there is a point X incident with α that is neither incident with a nor with b.
(25 points)

Problem 2: In Figure 2.5A on page 22 of the textbook, suppose that the point F is at infinity, so that the line PQ is parallel to the line AB. Show that C is the midpoint between A and B.
(Hint: Again, this problem does not fit perfectly into the framework of projective geometry considered here. There are at least two solutions, one of which uses similar triangles.) (25 points)

Problem 3: As in Problem 3 of the midterm examination, we consider the real number line as the x-axis of the plane. If $A = 1$, $B = -1$, and $C = n$, a positive integer, show that the harmonic conjugate D, in other words, the point D with $H(AB, CD)$, is $D = 1/n$. (25 points)

Problem 4: Suppose that A, B, C, D are four distinct points on the line l, and consider the projectivity

$$ABCD \barwedge BADC$$

from l to l constructed in Theorem 1.63 on page 12 of the textbook. Suppose that X is any point on l, and that X' is the corresponding point under this projectivity. Show that, conversely, X is the point that corresponds to X' under this projectivity.
(Hint: Use Axiom 2.18 on page 15 of the textbook.) (25 points)

Due date: Tuesday, March 10, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.