## Projective Geometry

**Problem 1:** Suppose that $A, C, B, F$ are four collinear points with the property $H(AB, CF)$. In other words, $F$ is the harmonic conjugate of $C$ with respect to $A$ and $B$. Show that

$$|AC| : |CB| = |AF| : |BF|$$

(Hint: In Figure 2.5A on page 22 of the textbook, move the point $R$ to infinity and use similar triangles. $|AC|$ denotes the distance from $A$ to $C$. The formula therefore does not fit perfectly into the framework of projective geometry considered here.)                                     (25 points)

**Problem 2:** Suppose that $A, B, C$ are three collinear points, and that $C$ is the midpoint of $A$ and $B$. Find the harmonic conjugate $F$ of $C$ with respect to $A$ and $B$.                                     (25 points)

(Remark: Again, this problem uses distances and therefore does not fit perfectly into the framework of projective geometry considered here.)

**Problem 3:** A space configuration consists of points, lines, and planes in space with the following property: Each point is on the same number $a_{12}$ of lines and on the same number $a_{13}$ of planes; each line contains the same number $a_{21}$ of points and is on the same number $a_{23}$ of planes, and each plane contains the same number $a_{31}$ of points and the same number $a_{32}$ of lines. It is described by the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

where $a_{11}$ is the total number of points, $a_{22}$ is the total number of lines, and $a_{33}$ is the total number of planes. Find this matrix in the case of a tetrahedron.                                     (25 points)

**Problem 4:** If we intersect a tetrahedron with a plane $\alpha$ that does not go through its vertices, the intersections points of its lines and planes with $\alpha$ give a plane configuration in $\alpha$. Find the corresponding numbers $m$, $n$, $c$, and $d$ discussed on page 26 in Section 3.2 of the textbook, and describe the arising configuration.                                     (25 points)

Due date: Tuesday, March 3, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.