Projective Geometry

Problem 1: Suppose that α is a plane and that O is a point on α. We consider the pencil consisting of all lines in α through O. Let $x \barwedge x'$ be a projectivity from this pencil to itself. Suppose that a, b, and c are three distinct lines of this pencil that that satisfy $a = a'$, $b = b'$, and $c = c'$. Show that $x = x'$ for every line in the pencil. (25 points)

(Hint: This is the dual of Axiom 2.18 on page 15 in the textbook, which you should use for the proof.)

Problem 2: Suppose that PQR, $P'Q'R'$, $P''Q''R''$ are three triangles that are perspective from the point O. In other words, we assume that P' and P'' lie on the line OP, that Q' and Q'' lie on the line OQ, and that R' and R'' lie on the line OR. We assume that all these points and all lines connecting them are distinct. Consider the intersection points

$$D_1 = RQ \cdot R'Q' \qquad E_1 = PR \cdot P'R' \qquad F_1 = PQ \cdot P'Q'$$
$$D_2 = RQ \cdot R''Q'' \qquad E_2 = PR \cdot P''R'' \qquad F_2 = PQ \cdot P''Q''$$
$$D_3 = R'Q' \cdot R''Q'' \qquad E_3 = P'R' \cdot P''R'' \qquad F_3 = P'Q' \cdot P''Q''$$

According to Desargues' theorem (Theorem 2.32 on page 19 in the textbook), the points D_1, E_1, F_1 lie on one line l_1, D_2, E_2, F_2 lie on one line l_2, and D_3, E_3, F_3 lie on one line l_3. Show that l_1, l_2, and l_3 are concurrent. (25 points)

(Hint: Apply Theorem 2.31 on page 19 in the textbook to the triangles $D_1D_2D_3$ and $E_1E_2E_3$.)

Problem 3: The following picture shows two triangles that are perspective from the point O:
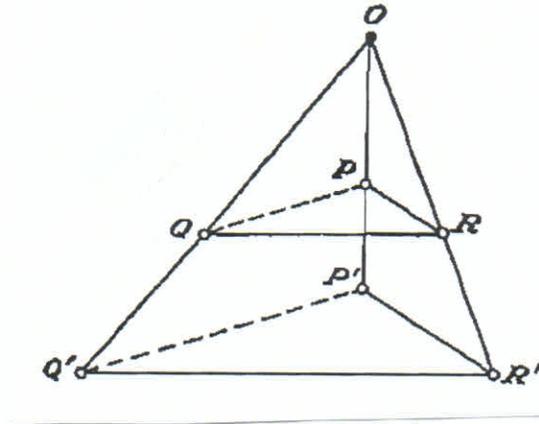
Discuss whether Desargues' theorem holds for these triangles.　　(25 points)

Problem 4: On a separate sheet of paper, draw a complete quadrangle in one colour and a complete quadrilateral in a different colour so that each side of the quadrangle contains exactly one vertex of the quadrilateral.　　(25 points)

Due date: Tuesday, February 11, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.