Memorial University of Newfoundland **Winter Semester 2020**
Yorck Sommerhäuser **MATH 3331: Sheet 2**

Projective Geometry

Problem 1: On a separate sheet of paper, draw a line with four consecutively marked points called O, G, E, and C. The distance from O to G should be ten centimetres long, the distance from G to E two centimetres, and the distance from E to C three centimetres. Then draw a complete quadrangle so that O and E are diagonal points of the quadrangle, while the two remaining lines of the quadrangle that would intersect in the third diagonal point intersect the given line in the points G and C. (25 points)

Problem 2: In the Fano plane, construct a complete quadrilateral whose three diagonal lines are concurrent, i.e., pass through the same point. (25 points)

Problem 3: From our axioms, deduce that the three diagonal lines of a complete quadrilateral are never concurrent. (Note that our seventh axiom, which is Axiom 2.17 on page 15 in the textbook, is not satisfied in the Fano plane, as we discussed in class.) (25 points)

Problem 4: Suppose that $PQRS$ is a quadrangle in a plane, so that no three of these points lie on a line. Suppose that $P'Q'R'S'$ is another quadrangle in the same plane, consisting of different points and having no sides in common. Suppose that there is a line o so that the six intersection points of corresponding sides of the quadrangles, for example the intersection point of PQ with $P'Q'$, all lie on o. Show that the lines PP', QQ', RR', and SS' all pass through the same point O.
(Hint: You may use the converse of Desargues' theorem, which is Theorem 2.31 on page 19 in the textbook.) (25 points)

Due date: Tuesday, February 11, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.