

Projective Geometry

Instructions: To solve the problems below, you can use the eight axioms in Section 2.1 on page 15 in the textbook, the exercises right below on that page, which have solutions in the back of the book and which we did in class, the consequences of the axioms stated in Section 2.2 on page 17 of the textbook, and the theorem on the two pages from the book by Veblen and Young that I handed out in class. Anything else that you want to use, you need to prove separately.

Problem 1: Suppose that α is a plane and that a is a line that is not in α . Show that a and α meet in at most one point. (25 points)

(Comment: It follows from the axioms that they indeed meet in exactly one point, but you do not need to show that.)

Problem 2: Suppose that a , b , and c are three distinct lines that are not coplanar, i.e., that are not contained in the same plane. Show that they meet in at most two points, i.e., that there are at most two points in which two of the three lines meet. (25 points)

Problem 3: Suppose that a , b , and c are three distinct lines that are coplanar, i.e., that are contained in the same plane. Show that they either are concurrent, i.e., that there is a point where all three lines meet, or that they meet in three distinct points, i.e., that there are distinct points P , Q , and R in each of which two of the three lines meet. (25 points)

Problem 4: On a separate sheet of paper, draw a complete quadrangle and indicate its diagonal points. (25 points)

Due date: Tuesday, January 21, 2020. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary arguments in full detail, using complete sentences. It is not necessary to copy down the problems again, to write down your student number, or to submit this sheet with your solution.