

## Introductory Number Theory

**Problem 1:** In the ring  $\mathbf{G}$  of Gaussian integers, determine which of the following divisibility relations hold.

1.  $(2 + 3i) \mid (5 - i)$  (6 points)
2.  $3 \mid (2 - i)(3 + i)$  (6 points)
3.  $(3 - 2i) \mid 26$  (6 points)
4.  $(4 - 5i) \mid (91 - 22i)$  (6 points)

(This is Problem 5 for Chapter 9 in the course notes.)

**Problem 2:** For  $\beta = 8 - 9i$  and  $\alpha = 3 + 5i$ , find  $\gamma$  and  $\delta$  satisfying the division algorithm in the ring  $\mathbf{G}$  of Gaussian integers. (25 points)

(This is Problem 3 for Chapter 9 in the course notes.)

**Problem 3:** In the ring  $\mathbf{G}$  of Gaussian integers, find a greatest common divisor of  $\alpha = 531 + 582i$  and  $\beta = 54 - 101i$ . (25 points)

(This problem is taken from page 97 in Chapter 9 in the course notes.)

**Problem 4:** Factor the Gaussian integer  $304 + 228i$  completely into primes. (26 points)

(This is part of Problem 9 for Chapter 9 in the course notes.)

Due date: Monday, November 20, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.