Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 3370: Sheet 4**

Introductory Number Theory

Problem 1: Given the number 2492, double the units digit and subtract it from the number formed by the other digits. We get $249 - 2 \times 2 = 245$. Repeating this algorithm we get $24 - 2 \times 5 = 14$. Since 14 is clearly divisible by 7, the original number 2492 must be divisible by 7. Prove this rule for checking divisibility by 7. (25 points)
(This is Problem 10 for Chapter 3 in the course notes.)

Problem 2: Dirichlet proved that there are always infinitely many primes in any arithmetic sequence $a, a + d, a + 2d, \ldots$, where $(a, d) = 1$. Prove that there are infinitely many primes in the arithmetic sequence $3, 7, 11, 15, \ldots$.
(Hint: Consider the numbers of the form $4p_1p_2 \ldots p_r - 1$, where each p_i is prime and of the form $4k - 1$.) (25 points)
(This is Problem 15 for Chapter 3 in the course notes.)

Problem 3: Let $f(x) = 375x^5 - 131x^4 + 15x^2 - 435x - 2$. Find the remainder when $f(97)$ is divided by 11. (25 points)
(This is Problem 3 for Chapter 4 in the course notes. The answer is indeed at the end of the notes, but, as always, you are supposed to justify your answer.)

Problem 4: Suppose that p is an odd prime. Show that

1. $1^{p-1} + 2^{p-1} + \ldots + (p - 1)^{p-1} \equiv -1 \pmod{p}$. (13 points)
2. $1^p + 2^p + \ldots + (p - 1)^p \equiv 0 \pmod{p}$. (12 points)

(This is Problem 14 for Chapter 4 in the course notes.)

Due date: Wednesday, October 11, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.