Introductory Number Theory

Problem 1: For $a = 4409$ and $b = 519$, find the greatest common divisor and find x and y such that $ax + by = \gcd(a,b)$ by making a four-column table with entries c_n, d_n, r_n, and q_n as on page 24 of the course notes.   (25 points)
(This is Problem 11 (c) for Chapter 2 in the course notes. Similar questions are not unlikely to be on the midterm exam.)

Problem 2: Let $\{f_n\}$ be the Fibonacci sequence, defined in Chapter 1 of the notes as well as in Problem 4 on Sheet 1. Prove that in finding (f_{n+2}, f_{n+1}) by the Euclidean algorithm, n divisions are necessary.   (25 points)
(This is Problem 15 for Chapter 2 in the course notes.)

Problem 3:

1. Let a be a positive integer different from 1, and s and t positive integers. Prove that $(a^t - 1)$ divides $(a^s - 1)$ if and only if t divides s. (15 points)

2. Prove that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$ for any positive integer a with $a \neq 1$. (10 points)

(This is Problem 16 for Chapter 2 in the course notes. At the end, the notes contain a hint for this problem.)

Problem 4: A man sold his sheep for \$180 each and his cows for \$290 each. He received a total of \$2890. How many cows did he sell?   (25 points)
(This is Problem 39 for Chapter 2 in the course notes.)

Due date: Monday, September 25, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.