Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 3370: Sheet 1**

Introductory Number Theory

Problem 1:

1. Prove that
$$x^{2n} - x^{2n-1} + \ldots + x^2 - x + 1 = \frac{x^{2n+1} + 1}{x + 1}.$$
(10 points)

2. If $2^a + 1$ is prime, prove that $a = 2^n$ for some $n \in N$. (Hint: Part 1 gives a formula for factoring $x^m + 1$ for m odd.) (15 points)

(This is Problem 2 for Chapter 1 in the course notes.)

Problem 2: Let $F_n = 2^{2^n} + 1$.

1. Prove using mathematical induction that
$$F_n = 2 + F_0 \cdot F_1 \cdots F_{n-1}$$
(12 points)

2. Prove that, $m \neq n$, there is no natural number $d \neq 1$ that divides both F_m and F_n. (13 points)

(This is Problem 10 for Chapter 1 in the course notes. The second part is known as Goldbach's theorem.)

Problem 3: Prove that no perfect number is a power of a prime. (25 points)
(This is Problem 8 for Chapter 1 in the course notes.)

Problem 4: The Fibonacci numbers are defined recursively by the conditions $f_1 = 1 = f_2$ and
$$f_n := f_{n-1} + f_{n-2}$$
if $n \geq 3$. Prove Binet's formula
$$f_n = \frac{\alpha^n - \beta^n}{\sqrt{5}}$$
for all $n \geq 1$, where $\alpha := \frac{1+\sqrt{5}}{2}$ and $\beta := \frac{1-\sqrt{5}}{2}$. (25 points)
(This is Problem 12 for Chapter 1 in the course notes.)

Due date: Monday, September 18, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.