Memorial University of Newfoundland
Yorck Sommerhäuser

Fall Semester 2017
MATH 2051: Sheet 8

Linear Algebra II

Problem 1: Consider the matrix

$$A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$$

1. Find the characteristic polynomial $p(\lambda)$. (5 points)
2. Factor the characteristic polynomial into linear factors and determine the eigenvalues. (5 points)
3. For each of the eigenvalues, find a basis of the corresponding eigenspace. (5 points)
4. Find an invertible matrix P such that $D := P^{-1}AP$ is diagonal. Without computing the matrix product $P^{-1}AP$, state what D is and justify your statement. (5 points)
5. Compute the matrix product $P^{-1}AP$ explicitly and verify your last assertion. (5 points)

Problem 2: According to Equation (5) on page 30 in Section 1.3 of the textbook, the (i, j)-entry of a matrix product $C = AB$ is

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{ir}b_{rj}$$

where the entries of the matrices can be complex numbers. Using this formula, prove that

$$\bar{C} = \bar{A}\bar{B}$$

where, as defined on page 315 in Section 5.3 of the textbook, the matrix $\bar{A}$ is the matrix whose entries are the complex conjugates of the entries of A. Note that this exercise proves Theorem 5.3.2 (c) in the textbook. (25 points)

Problem 3: Consider the matrix

$$A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

where $b \neq 0$. Find the eigenvalues of A, and for each eigenvalue, find a basis of the corresponding eigenspace. (25 points)

Problem 4: Consider a rotation around the origin with an angle ϕ in the plane.

1. Explain why this rotation is a linear transformation. (5 points)
2. Explain why this rotation will rotate the standard basis vector $(1, 0)$ into the vector $(\cos(\phi), \sin(\phi))$. (4 points)
3. Find the image of the standard basis vector $(0, 1)$ under this rotation. (5 points)
4. Find the matrix representation A of the rotation with respect to the standard basis. (5 points)
5. Find the eigenvalues of A. For each eigenvalue, find a basis of the corresponding eigenspace. (5 points)
6. Determine for which angles ϕ the eigenvalues are real. (1 point)

(Hint: Problem 3 can be used for the solution of Problem 4.)

Due date: Wednesday, November 15, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your names on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.