Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 2051: Sheet 7**

Linear Algebra II

Problem 1: State the so-called 'rational zero test' from a precalculus textbook of your choice. The test may also have a slightly different name there. Do this in the following way: Write down the statement of this test, the authors and the title of the textbook that you are using, as well as the library of congress classification. An example of such a classification is QA331.3.F34 2007. It can be determined from the catalogue of the university library or the library of congress. Sometimes, it can be found at the beginning of the book. In addition, submit a photocopy of the page on which the rational zero test is stated in that book. If you do not own a precalculus textbook, find one in the university library. It is not sufficient to submit a statement of the rational zero test from some internet page or some set of unpublished lecture notes found on the internet. Also not sufficient is to use the very brief discussion of this test in our textbook. (25 points)

Problem 2:

1. For the matrix
$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$
find the characteristic polynomial $p(\lambda)$. (5 points)

2. Factor the characteristic polynomial into linear factors and determine the eigenvalues. Explain how the rational zero test can be used in this process. (10 points)

3. For each of the eigenvalues, find a basis of the corresponding eigenspace. (10 points)

Problem 3: We continue the preceding problem and use the notations from there.

1. Decide whether A is diagonalizable. (1 point)

2. Find a basis B of R^3 that consists only of eigenvectors of A. (1 point)

3. Find the transition matrix P from the basis B to the standard basis S. (8 points)

4. Compute the inverse P^{-1} of P, which is the transition matrix from the standard basis S to the basis B. (10 points)

5. Compute the matrix $P^{-1}AP$. (5 points)

Problem 4: Decide whether the matrix

$$A = \begin{bmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{bmatrix}$$

is diagonalizable. (As always, justify your assertions completely.) (25 points)

Due date: Monday, November 6, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your names on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.