Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 2051: Sheet 6**

Linear Algebra II

Problem 1: Let $T : R^2 \to R^2$ be the linear operator defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}$$

1. Let $B = \{\mathbf{e}_1, \mathbf{e}_2\}$ be the standard basis
$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
of R^2. Find $[T]_{B,B}$. (5 points)
2. Let $B' = \{\mathbf{u}_1, \mathbf{u}_2\}$ be the basis with
$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$
Find $[T]_{B',B'}$. (20 points)

(You do not need to show that B' is indeed a basis.)

Problem 2: We use the same notation as in Problem 1.

1. Find the transition matrices $P_{B\to B'}$ and $P_{B'\to B}$. (10 points)
2. Show that they are inverses of each other. (5 points)
3. Compute the matrix product $P_{B\to B'}[T]_{B,B}P_{B'\to B}$ and compare the result with $[T]_{B',B'}$. (10 points)

Problem 3: We denote by P_1 and P_2 the vector space of polynomials of degree at most 1 and degree at most 2, respectively. Consider the linear transformation $T_1 : P_1 \to P_2$ defined by

$$T_1(p(x)) = xp(x)$$

and the linear transformation $T_2 : P_2 \to P_2$ defined by

$$T_2(p(x)) = p(2x + 1)$$

Let $B = \{1, x\}$ and $B' = \{1, x, x^2\}$ be the standard bases of P_1 and P_2.

1. Find $[T_1]_{B',B}$ and $[T_2]_{B',B'}$. (10 points)

2. Compute $T_2 \circ T_1$ directly and then find $[T_2 \circ T_1]_{B',B}$. (10 points)

3. Find the matrix product $[T_2]_{B',B'}[T_1]_{B',B}$ and confirm that it agrees with the result of the previous part. (5 points)

Problem 4: Consider a linear transformation $T : V \to V$ of a finite-dimensional vector space V, and let B be a basis of V.

1. Show that T is the identity transformation if and only if $[T]_{B,B}$ is the identity matrix. (15 points)

2. Using the problems on this exercise sheet, find an example of a vector space V and bases B and B' of V, so that $[T]_{B',B}$ is not the identity matrix, even though T is the identity transformation. (10 points)

Due date: Monday, October 30, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your names on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.