Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 2051: Sheet 5**

Linear Algebra II

Problem 1: Consider the linear transformation

$$T : R^2 \to R^3, \ (x, y) \mapsto (x - y, y - x, 2x - 2y)$$

(You do not need to show that T is linear; this will follow from the first part below.)

1. Find a 3×2-matrix A such that, after viewing vectors as column vectors, we have
$$T(x, y) = A \begin{bmatrix} x \\ y \end{bmatrix}$$
(8 points)

2. Find the kernel of T by determining a basis of the kernel. (If the kernel is the zero vector space, the empty set is a basis.) (8 points)

3. Determine whether T is injective, surjective, or bijective, respectively. (In each case, as always, justify your claims.) (9 points)

Problem 2: Consider the linear transformation $T : P_2 \to P_2$ defined by

$$T(p(x)) = p(x + 1)$$

Determine whether T is injective, surjective, or bijective, respectively. (In each case, as always, justify your claims. You do not need to show that T is linear.) (25 points)

Problem 3: Suppose that $T : V \to W$ is a bijective linear transformation between the vector spaces V and W. Prove that the inverse transformation $T^{-1} : W \to V$ is linear. (25 points)

Problem 4: Consider the linear transformation

$$T : R^2 \to R^2, \ (x, y) \mapsto (x + y, x - y)$$

(You do not need to show that T is linear; this will follow from the first part below.)

1. Find a 2×2-matrix B such that, after viewing vectors as column vectors, we have

$$T(x, y) = B \begin{bmatrix} x \\ y \end{bmatrix}$$

(8 points)

2. Show that T is bijective. (8 points)

3. Find the inverse $T^{-1} : R^2 \to R^2$ by giving an explicit formula for $T^{-1}(x, y)$. (9 points)

Due date: Monday, October 23, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your names on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.