Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 2051: Sheet 3**

Linear Algebra II

Problem 1: Decide whether the polynomials

$$1+x, \qquad 1-x, \qquad 1-x^2, \qquad 1-x^3$$

form a basis of P_3, the space of polynomials of degree at most 3. (25 points)

Problem 2: Consider the vectors $\mathbf{u}_1 = (1,-1)$ and $\mathbf{u}_2 = (1,1)$.

1. Show that $S = \{\mathbf{u}_1, \mathbf{u}_2\}$ is a basis of R^2. (12 points)
2. For $\mathbf{w} = (1,0)$, find the coordinate vector $(\mathbf{w})_S$ with respect to this basis. (13 points)

Problem 3: Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for a vector space V. If we define

$$\mathbf{u}_1 := \mathbf{v}_1, \qquad \mathbf{u}_2 := \mathbf{v}_1 + \mathbf{v}_2, \qquad \mathbf{u}_3 := \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$$

show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is also a basis for V. (25 points)

Problem 4: Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a basis for a vector space V. Show that the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ are linearly independent in V if and only if their coordinate vectors $(\mathbf{u}_1)_S, (\mathbf{u}_2)_S, (\mathbf{u}_3)_S$ are linearly independent in R^4.

(Remark: Make sure that treat both directions, the 'if'-part and the 'only if' part.) (25 points)

Due date: Monday, October 2, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your names on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.