

Linear Algebra II

Problem 1: Express the polynomial

$$p = 6 + 11x + 6x^2$$

as a linear combination of the polynomials $p_1 = 2 + x + 4x^2$, $p_2 = 1 - x + 3x^2$, and $p_3 = 3 + 2x + 5x^2$. (25 points)

Problem 2: Consider the functions $f(x) = \cos^2(x)$ and $g(x) = \sin^2(x)$.

1. Decide whether $\cos(2x)$ is a linear combination of f and g . (12 points)
2. Decide whether $1 + x^2$ is a linear combination of f and g . (13 points)

Problem 3: Determine all values of k for which the following matrices are linearly independent in M_{22} :

$$\begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ k & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

(25 points)

Problem 4: Consider the functions $f_1(x) = e^x$, $f_2(x) = xe^x$, and $f_3(x) = x^2e^x$.

1. Compute the Wronskian determinant of these functions. (20 points)
2. Using the Wronskian determinant just computed, decide whether these functions are linearly independent in $F(-\infty, \infty)$. (5 points)

(Hint: If you claim that a function is not zero, make sure that this is really the case, for example by plugging in a value.)

Due date: Monday, September 25, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your names on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.