Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 2051: Sheet 1**

Linear Algebra II

Problem 1: Consider the vector space $V = F(-\infty, \infty)$ from Example 6 of Section 4.1 on page 187 of the textbook and covered in class. Complete the treatment given there and in class by discussing for each of the ten vector space axioms listed in Definition 1 of Section 4.1 three pages earlier why it holds.
(25 points)

Problem 2: Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$:

$$\mathbf{u} + \mathbf{v} := (u_1 + v_1 + 1, u_2 + v_2 + 1), \qquad k\mathbf{u} = (ku_1, ku_2)$$

1. Prove that Axiom 4 in Definition 1 of Section 4.1 holds by showing that $\mathbf{0} = (-1, -1)$ is a zero vector. (7 points)
2. Show that Axiom 5 holds. (8 points)
3. Find two vector space axioms that fail to hold. (10 points)

(Note: This problem is based on Exercise 2 for Section 4.1 in the textbook.)

Problem 3: Do Exercise 22 in Section 4.1 by copying the proof given there and adding the justifications asked for. (25 points)

Problem 4: As in Problem 1, consider the vector space $V = F(-\infty, \infty)$.

1. Decide whether the subset
$$U := \{\mathbf{f} \in V \mid \mathbf{f}(1) = 0\}$$
of functions that take the value 0 at 1 is a subspace of V. (13 points)
2. Decide whether the subset
$$U := \{\mathbf{f} \in V \mid \mathbf{f}(0) = 1\}$$
of functions that take the value 1 at 0 is a subspace of V. (12 points)

Due date: Monday, September 18, 2017. Work in groups of three students. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.