Memorial University of Newfoundland **Fall Semester 2017**
Yorck Sommerhäuser **MATH 2051: Sheet 10**

Linear Algebra II

Problem 1: Decide whether the matrix

$$A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

is orthogonal.

Problem 2: Prove that any orthogonal 2×2-matrix A has the form

$$A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

if $\det(A) = 1$ and

$$A = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

if $\det(A) = -1$.

(Hint: Start with a general 2×2-matrix A, and use the fact that the column vectors form an orthonormal basis of R^2.)

Problem 3: Consider the matrix

$$A = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

appearing in the preceding problem.

1. Find its eigenvalues and for each eigenvalue a basis of the corresponding eigenspace.
2. Use these eigenvectors to explain that A describes a reflection at a line through the origin. Determine the angle that this line makes with the x-axis.

Problem 4: Find a QR-decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

i.e., write A in the form $A = QR$, where Q is orthogonal and R is upper triangular.

Due date: There is no due date. Completion of these problems is voluntary. The solutions will be neither collected nor marked. However, the problems constitute valuable practice for the final exam.