Memorial University of Newfoundland **Winter Semester 2020**
Yorck Sommerhäuser **MATH 6329: Sheet 6**

Hopf Algebras

Problem 1: Show that the antipode of the Taft algebra T, considered in Problem 2 on Sheet 2, has order exactly $2n$.
(Remark: You need to show that the order is not smaller. This exercise shows that there are finite-dimensional Hopf algebras with antipodes of arbitrary even order.) (20 points)

Problem 2: Suppose that H is a finite-dimensional Hopf algebra whose antipode has odd order. Show that this order is 1. (25 points)

Problem 3: If H is a finite-dimensional Hopf algebra and $\Lambda \in H$ and $\lambda \in H^*$ are left integrals satisfying $\lambda(\Lambda) = 1$, show that the antipode can be computed from the integrals via the formula

$$S(h) = \Lambda_{(1)}\lambda(h\Lambda_{(2)})$$

(15 points)

Problem 4: Suppose that H is a finite-dimensional commutative semisimple Hopf algebra over the algebraically closed field K. Show that there is a finite group G so that $H \cong K[G]^*$. (40 points)

Due date: There is no due date. The completion of these problems is voluntary. The solutions will not be collected and not be marked, unless explicitly requested otherwise.