Memorial University of Newfoundland **Winter Semester 2020**
Yorck Sommerhäuser **MATH 6329: Sheet 5**

Hopf Algebras

Problem 1: Suppose that G is a finite group, and consider the dual group algebra $K[G]^*$ discussed in Problem 1 on Sheet 3. Find the left and the right integrals in $K[G]^*$ and decide whether $K[G]^*$ is unimodular. (20 points)

Problem 2: Let T be the Taft algebra, considered in Problem 2 on Sheet 2.

1. Find the left and the right integrals of T. (20 points)
2. Find the left and the right modular function of T, and decide whether T is unimodular. (5 points)

Problem 3: Suppose that A is a Frobenius algebra and that $\varepsilon : A \to K$ is an algebra homomorphism. Extending the definition for Hopf algebras, we say that $\Lambda \in A$ is a left integral if we have $a\Lambda = \varepsilon(a)\Lambda$ for all $a \in A$. Show that the subspace of left integrals in A is one-dimensional. (Remark: Frobenius algebras are finite-dimensional by definition.) (25 points)

Problem 4: Suppose that H is a finite-dimensional Hopf algebra and that $\Lambda \in H$ is a left integral.

1. Show that $\Delta(\Lambda) = S^2(\Lambda_{(2)})a^R \otimes \Lambda_{(1)}$. (25 points)
2. Deduce from this equation that $S^2(\Lambda) = \alpha^L(a^L)\Lambda$. (5 points)

(Remark: Here α^L, α^R, a^L, and a^R are the modular functions and elements, respectively.)

Due date: There is no due date. The completion of these problems is voluntary. The solutions will not be collected and not be marked, unless explicitly requested otherwise.