

Hopf Algebras

Problem 1: Show that the antipode of the Taft algebra T , considered in Problem 2 on Sheet 2, has order exactly $2n$.

(Remark: You need to show that the order is not smaller. This exercise shows that there are finite-dimensional Hopf algebras with antipodes of arbitrary even order.) (5 points)

Problem 2: Suppose that H is a finite-dimensional Hopf algebra whose antipode has odd order. Show that this order is 1. (4 points)

Problem 3: If H is a finite-dimensional Hopf algebra and $\Lambda \in H$ and $\lambda \in H^*$ are left integrals satisfying $\lambda(\Lambda) = 1$, show that the antipode can be computed from the integrals via the formula

$$S(h) = \Lambda_{(1)}\lambda(h\Lambda_{(2)})$$

(3 points)

Problem 4: Suppose that H is a finite-dimensional commutative semisimple Hopf algebra over the algebraically closed field K . Show that there is a finite group G so that $H \cong K[G]^*$. (8 points)

Due date: Thursday, March 3, 2016. Please write your solution on letter-sized paper, and write your name on your solution. Give all your computations in complete detail, and explain these computations in English, using complete sentences. It is not necessary to copy down the problems again, and it is also not necessary to submit this sheet with your solution.