Memorial University of Newfoundland **Fall Semester 2024**
Yorck Sommerhäuser **MATH 6329: Sheet 5**

Hopf Algebras

Problem 1: Show that the antipode of the Taft algebra T, considered in Problem 2 on Sheet 2, has order exactly $2n$.
(Remark: You need to show that the order is not smaller. This exercise shows that there are finite-dimensional Hopf algebras with antipodes of arbitrary even order.) (25 points)

Problem 2: Suppose that H is a finite-dimensional Hopf algebra whose antipode has odd order. Show that this order is 1. (20 points)

Problem 3: If H is a finite-dimensional Hopf algebra and $\Lambda \in H$ and $\lambda \in H^*$ are left integrals satisfying $\lambda(\Lambda) = 1$, show that the antipode can be computed from the integrals via the formula

$$S(h) = \Lambda_{(1)} \lambda(h\Lambda_{(2)})$$

(15 points)

Problem 4: Suppose that H is a finite-dimensional commutative semisimple Hopf algebra over the algebraically closed field K. Show that there is a finite group G so that $H \cong K[G]^*$. (40 points)

Due date: Thursday, October 17, 2024. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.