Memorial University of Newfoundland **Fall Semester 2024**
Yorck Sommerhäuser **MATH 6329: Sheet 4**

Hopf Algebras

Problem 1: Suppose that G is a finite group, and consider the dual group algebra $K[G]^*$ discussed in Problem 1 on Sheet 3. Find the left and the right integrals in $K[G]^*$ and decide whether $K[G]^*$ is unimodular. (20 points)

Problem 2: Let T be the Taft algebra, considered in Problem 2 on Sheet 2.

1. Find the left and the right integrals of T. (20 points)
2. Find the left and the right modular function of T, and decide whether T is unimodular. (5 points)

Problem 3: Suppose that A is a Frobenius algebra and that $\varepsilon : A \to K$ is an algebra homomorphism. Extending the definition for Hopf algebras, we say that $\Lambda \in A$ is a left integral if we have $a\Lambda = \varepsilon(a)\Lambda$ for all $a \in A$. Show that the subspace of left integrals in A is one-dimensional. (Remark: Frobenius algebras are finite-dimensional by definition.) (25 points)

Problem 4: Suppose that H is a finite-dimensional Hopf algebra and that $\Lambda \in H$ is a left integral.

1. Show that $\Delta(\Lambda) = S^2(\Lambda_{(2)})a^R \otimes \Lambda_{(1)}$. (25 points)
2. Deduce from this equation that $S^2(\Lambda) = \alpha^L(a^L)\Lambda$. (5 points)

(Remark: Here α^L, α^R, a^L, and a^R are the modular functions and elements, respectively.)

Due date: Tuesday, October 8, 2024. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.