Memorial University of Newfoundland **Fall Semester 2024**
Yorck Sommerhäuser **MATH 6329: Sheet 10**

Hopf Algebras

Problem 1: The symmetric group S_3 on three letters has a unique irreducible character of degree 2. Find the third Frobenius-Schur indicator $\nu_3(\chi)$ of this character. (20 points)

Problem 2: As we saw in Problem 4 on Sheet 9, the Kac-Palyutkin algebra, which was defined in Problem 1 on Sheet 9, also has a unique irreducible character of degree 2. Find the second Frobenius-Schur indicator $\nu_2(\chi)$ of this character. (30 points)

Problem 3: Suppose that H is a finite-dimensional semisimple Hopf algebra over a field of characteristic zero. The dual space $D(H)^*$ of its Drinfel'd double can be identified with $H \otimes H^*$ by considering $h \otimes \varphi \in H \otimes H^*$ as the linear form

$$(h \otimes \varphi)(\varphi' \otimes h') = \varphi'(h)\varphi(h')$$

If $\chi \in \mathrm{Ch}(D(H)) \subset D(H)^*$ is a character, we can therefore write it in the form

$$\chi = \sum_{i=1}^{r} h_i \otimes \varphi_i$$

Show that $\Phi(\chi) = \sum_{i=1}^{r} \varphi_i \otimes h_i$. (20 points)

Problem 4: Suppose that G is a finite group and that $H = K[G]$ is its group ring over the field K. For $g \in G$, let $C(g) \subset G$ be the centralizer of g, and let W be a $C(g)$-module.

1. Show that the induced module $V := H \otimes_{K[C(g)]} W$ becomes a module for the Drinfel'd double $D(H)$ if we define

$$(p_{g'} \otimes h').(h \otimes_{K[C(g)]} w) = \delta_{g',h'hgh^{-1}h'^{-1}} h'h \otimes_{K[C(g)]} w$$

2. Show that V is irreducible if W is irreducible. (30 points)

Due date: Tuesday, November 26, 2024. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.