Memorial University of Newfoundland **Winter Semester 2017**
Yorck Sommerhäuser **MATH 4321: Sheet 3**

Group Theory

Problem 1: The symmetric group S_3 on three letters contains the six elements

$$\mathrm{id}, (1,2), (1,3), (2,3), (1,2,3), (1,3,2)$$

Write each of these six elements as a word in the elements $(1,2)$ and $(1,3)$. (Hint: The definition of a word can be found on page 23 of the textbook. The set X mentioned there is $X = \{(1,2), (1,3)\}$.) (15 points)

Problem 2: Inside the group $G := S_3$, consider the subgroups $A := \langle(1,2,3)\rangle$ and $B := \langle(1,2)\rangle$.

1. For the equivalence relation described in Problem 4 of Sheet 1, list explicitly all equivalence classes. (5 points)
2. Describe the bijection between the set of these equivalence classes and the set AB. (2 points)
3. Find $|AB|$, $|A \cap B|$, and $|A \times B|$ in this example. (3 points)

Problem 3: The alternating group A_4 on four letters is the subgroup of S_4 that consists of the even permutations. It is therefore the kernel of the sign homomorphism.

1. Explain how the homomorphism theorem and the fact that S_4 contains $4! = 24$ elements imply that A_4 contains 12 elements. (5 points)
2. Besides the entire group and the subgroup that consists only of the identity, A_4 contains eight subgroups. Determine these subgroups. On a separate, otherwise empty sheet of paper, draw a picture of these ten subgroups with the entire group on top and the group consisting of the identity at the bottom. Indicate by arrows which subgroups are contained in which subgroups. Make sure that subgroups of a similar type are closely together on your sheet. Justify your analysis on separate pages. (30 points)

(Remark: The subgroups of similar type mentioned above are conjugate to one another, i.e., arise from one another by an inner automorphism. This can be seen, for example, with the help of Problem 1 on Sheet 1.)

Problem 4: Suppose that A, B, and C are subgroups of a group G.

1. If $B \subset C$, show the so-called Dedekind law

$$(AB) \cap C = (A \cap C)B$$

for the complex products. (20 points)

2. Using the previous problem, find subgroups A, B, and C of $G = A_4$ that do not satisfy

$$(AB) \cap C = (A \cap C)(B \cap C)$$

(Note that then we cannot have $B \subset C$.) (20 points)

Due date: Monday, February 6, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.