Memorial University of Newfoundland **Winter Semester 2017**
Yorck Sommerhäuser **MATH 4321: Sheet 2**

Group Theory

Problem 1: Besides the subgroup consisting only of the unit element and the group itself, the symmetric group S_3 on three letters contains four subgroups. Find these four subgroups and determine which of them are normal.
(Hint: Problem 1 on Sheet 1 can be helpful. As always, give a detailed justification of your answer.) (25 points)

Problem 2: Consider the subset

$$V := \{\mathrm{id}, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$$

of the symmetric group S_4 on four letters.

1. Make a multiplication table for V. (10 points)
2. Use this multiplication table to show that V is a subgroup of S_4.(5 points)
3. Show that V is a normal subgroup of S_4.
   (Hint: Again, Problem 1 on Sheet 1 can be helpful.) (10 points)

Problem 3: Suppose that S is a subgroup of a group G. If $[G : S] = 2$, show that S is a normal subgroup. (25 points)

Problem 4:

1. Show that the composition of two group homomorphisms between not necessarily equal groups is again a group homomorphism. (2 points)
2. Show that the automorphisms of a group G form a subgroup of the symmetric group S_G of all bijective mappings from G to itself. This subgroup is denoted by $\mathrm{Aut}(G)$. (8 points)
3. Show that the inner automorphisms of G form a normal subgroup of the automorphism group $\mathrm{Aut}(G)$. (15 points)

Due date: Monday, January 30, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.