Memorial University of Newfoundland
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Winter Semester 2017
MATH 4321: Sheet 1

Group Theory

Problem 1: In the symmetric group S_n on n letters, consider an arbitrary permutation σ and a cycle $(i_1, i_2, \ldots, i_k)$ of length k. Show that

$$\sigma \circ (i_1, i_2, \ldots, i_k) \circ \sigma^{-1} = (\sigma(i_1), \sigma(i_2), \ldots, \sigma(i_k))$$

(10 points)

Problem 2: A cycle of length 2 is called a transposition. Show that every cycle of length k in S_n is the product of $k - 1$ transpositions. (20 points)

Problem 3: Show that every permutation $\sigma \in S_n$ can be written as a product of disjoint cycles. (30 points)

Problem 4: Suppose that G is a group and that A and B are nonempty subsets of G. We define the complex product of A and B as

$$AB := \{ab \mid a \in A, b \in b\}$$

1. Show that the complex product defines an associative multiplication on the set of nonempty subsets of G, i.e., that we have $(AB)C = A(BC)$. (2 points)
2. On the Cartesian product $A \times B$, we define the relation $\sim$ by

   $$(a, b) \sim (a', b') :\Leftrightarrow ab = a'b'$$

   Show that this relation is an equivalence relation. (3 points)
3. Suppose from now on in this problem that A and B are finite subgroups of G. Show that each equivalence class of the relation above contains $|A \cap B|$ elements. (12 points)
4. Show that

   $$|AB| = \frac{|A||B|}{|A \cap B|}$$

   (23 points)

Due date: Monday, January 23, 2017. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.

Change of syllabus: Office hours will be Monday and Friday from 1:00 pm to 3:00 pm.