Memorial University of Newfoundland **Fall Semester 2016**
Yorck Sommerhäuser **MATH 3210: Sheet 7**

Introduction to Complex Analysis

Problem 1: Explain how the integral

$$\int_1^i z^5 dz$$

is defined using contour integrals and find its value. (25 points)

Problem 2: Let C be the ellipse given by the equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

oriented positively, i.e., counterclockwise. Find the contour integral

$$\int_C \frac{1}{z} dz$$

(25 points)

Problem 3: Without computing the integral explicitly, use the standard estimate to show that the contour integral

$$\int_C \frac{z^3 + z + 2}{z^7 + z^3 + 1} dz$$

where C is a quarter-circle of radius 2 around the origin in the first quadrant, cannot be larger than $12\pi/119$. (25 points)

Problem 4: Let $D \subset \mathbb{C}$ be a domain in the Argand plane, and $f : D \to \mathbb{C}$ be a continuous function. If $\gamma : [c, d] \to D \cap \mathbb{R}$ is a contour that does not leave the real line, show that

$$\int_\gamma f(z)dz = \int_a^b f(x)dx$$

where $a = \gamma(c)$ and $b = \gamma(d)$. In other words, show that the contour integral and the Riemann integral from Calculus II give the same result in this situation, where, as discussed in class, the integral on the right-hand side is defined by integrating real and imaginary parts separately. (25 points)

Due date: Tuesday, November 8, 2016. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.