Memorial University of Newfoundland **Fall Semester 2016**
Yorck Sommerhäuser **MATH 3210: Sheet 2**

Introduction to Complex Analysis

Problem 1: The equation $z^3 = 27i$ has three complex solutions, the three roots of $27i$. For each of these roots, find their real and imaginary part. In the Argand plane, i.e., the complex plane, draw a picture that shows these three roots. (25 points)

Problem 2: If $z = (\ln 2, \frac{7\pi}{6}) = \ln 2 + \frac{7\pi}{6}i$, compute explicitly the complex number e^z and find its real and imaginary parts. Simplify your answer so that no exponentials, logarithms, or trigonometric functions appear in the answer. (15 points)

Problem 3: Find the domain of

$$f(z) = \frac{1}{z^2 + 6z - iz - 6i}$$

and find explicitly the real and imaginary parts of f that are determined by the equation

$$f(z) = u(x, y) + iv(x, y)$$

(cf. page 38 of the textbook). (25 points)

Problem 4: Consider the maps

$$f(z) = \frac{z-1}{z+1} \qquad \text{and} \qquad g(z) = \frac{1+z}{1-z}$$

1. Find the domain of both f and g. (5 points)
2. Show that f maps the domain of f to the domain of g and vice versa. (5 points)
3. Show that $g(f(z)) = z$ and $f(g(z)) = z$ whenever these expressions are defined. Determine where they are defined. (5 points)
4. Find the image of the imaginary axis under the map f. (20 points)

(Hint: Do the second and the third part simultaneously.)

Due date: Tuesday, September 27, 2016. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.