Memorial University of Newfoundland **Fall Semester 2016**
Yorck Sommerhäuser **MATH 3210: Sheet 11**

Introduction to Complex Analysis

Problem 1: Find the Maclaurin series for the function

$$f(z) = \frac{1}{(z+1)^3}$$

in closed form, and determine its radius of convergence.
(Hint: Use differentiation of power series, starting from a variant of the geometric series.)

Problem 2: Find the Maclaurin series for the function

$$f(z) = \mathrm{Log}(1+z)$$

in closed form, and determine its radius of convergence.
(Hint: Start from the Maclaurin series of the derivative. The symbol Log denotes the principal branch of the logarithm function.)

Problem 3: Consider the function

$$f(z) := \frac{z+1}{z^2 - 3z - 10}$$

If C is a circle of radius 6 around the origin, oriented counterclockwise, find

$$\int_C f(z)dz$$

(Hint: Use Cauchy's residue theorem.)

Problem 4: Recall the definition of the complex sine and cosine functions from Problem 1 on Sheet 8. The complex tangent, cotangent, secant, and cosecant functions are defined as in the real case

$$\tan(z) := \frac{\sin(z)}{\cos(z)} \qquad \cot(z) := \frac{\cos(z)}{\sin(z)}$$
$$\sec(z) := \frac{1}{\cos(z)} \qquad \csc(z) := \frac{1}{\sin(z)}$$

They are defined where the denominator is nonzero.

1. Show that all zeroes of the complex sine and cosine function are real.

2. Show that the origin is a simple pole of the cotangent function, and determine its residue there.

3. If C is a circle of radius 3 around the origin, oriented counterclockwise, find

$$\int_C \cot(z) dz$$

(Hint: The Maclaurin series for sine and cosine are considered as known; you can use them without further justification. A derivation is given in Example 3 and Example 4 of Section 64 of the textbook; we did a very similar computation in class at the beginning of the semester.)

Due date: There is no due date. The completion of this sheet is voluntary. The solutions will not be collected and will not be graded. However, these exercises provide valuable practice for the final exam.