Memorial University of Newfoundland **Fall Semester 2016**
Yorck Sommerhäuser **MATH 3210: Sheet 10**

Introduction to Complex Analysis

Problem 1: Consider the sequence $(f_n)_{n\in\mathbb{N}}$ that is formed by the functions $f_n(z) := z^n$.

1. Decide whether the sequence converges on the set $D := \{z \in \mathbb{C} \mid |z| < \frac{1}{2}\}$. If it converges, find its limit, i.e., the function to which it converges.
2. Decide whether the sequence converges uniformly on the set D.
3. Decide whether the sequence converges on the set $E := \{z \in \mathbb{C} \mid |z| < 1\}$. If it converges, find its limit, i.e., the function to which it converges.
4. Decide whether the sequence converges uniformly on the set E.

Problem 2: Prove or disprove that

$$\sum_{n=0}^{\infty} nz^n = \sum_{n=0}^{\infty} n^2 z^{2n}$$

for all complex numbers z satisfying $|z| < 1$.

Problem 3: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find a Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n z^n$$

about the origin that converges for all complex numbers z satisfying $4 < |z| < 6$. Make a table with the explicit values c_{-3}, c_{-2}, c_{-1}, c_0, c_1, c_2, c_3, and determine where precisely the Laurent series converges.

Problem 4: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find the Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-6)^n$$

about the point 6 that converges for points close to 6. Make a table with the explicit values c_{-3}, c_{-2}, c_{-1}, c_0, c_1, c_2, c_3, and determine where precisely the Laurent series converges.

Due date: There is no due date. The completion of this sheet is voluntary. The solutions will not be collected and will not be graded. However, these exercises provide valuable practice for the final exam.