Memorial University of Newfoundland **Fall Semester 2015**
Yorck Sommerhäuser **MATH 3210: Sheet 9**

Introduction to Complex Analysis

Problem 1: Consider the sequence $(f_n)_{n\in\mathbb{N}}$ that is formed by the functions $f_n(z) := z^n$.

1. Decide whether the sequence converges on the set $D := \{z \in \mathbb{C} \mid |z| < \frac{1}{2}\}$. If it converges, find its limit, i.e., the function to which it converges. (4 points)
2. Decide whether the sequence converges uniformly on the set D.(4 points)
3. Decide whether the sequence converges on the set $E := \{z \in \mathbb{C} \mid |z| < 1\}$. If it converges, find its limit, i.e., the function to which it converges. (4 points)
4. Decide whether the sequence converges uniformly on the set E.(4 points)

Problem 2: Prove or disprove that

$$\sum_{n=0}^{\infty} nz^n = \sum_{n=0}^{\infty} n^2 z^{2n}$$

for all complex numbers z satisfying $|z| < 1$. (17 points)

Problem 3: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find a Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n z^n$$

about the origin that converges for all complex numbers z satisfying $4 < |z| < 6$. Make a table with the explicit values c_{-3}, c_{-2}, c_{-1}, c_0, c_1, c_2, c_3, and determine where the Laurent series converges. (17 points)

Problem 4: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find the Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-6)^n$$

about the point 6 that converges for points close to 6. Make a table with the explicit values c_{-3}, c_{-2}, c_{-1}, c_0, c_1, c_2, c_3, and determine where precisely the Laurent series converges. (17 points)

Problem 5: Consider the function

$$f(z) = \begin{cases} \sin(z)/z & : z \neq 0 \\ 1 & : z = 0 \end{cases}$$

1. Determine where the function is continuous. (4 points)
2. Determine where the function is real differentiable, i.e., where the function $f(x)$ for $x \in \mathbb{R}$ is differentiable. (4 points)
3. Determine where the function is complex differentiable. (4 points)
4. Determine where the function is analytic and can therefore be developed into a convergent Taylor series. (4 points)

Problem 6: For the function

$$f(z) = \frac{\cos(z)}{z - \pi}$$

find the Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-\pi)^n$$

about the point π. Make a table with the explicit values c_{-3}, c_{-2}, c_{-1}, c_0, c_1, c_2, c_3, and determine where the Laurent series converges. (17 points)

Due date: Tuesday, November 24, 2015. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution. The hints given for Sheet 8 are also relevant for this sheet.