Memorial University of Newfoundland **Fall Semester 2015**
Yorck Sommerhäuser **MATH 3210: Sheet 8**

Introduction to Complex Analysis

Problem 1: Suppose that the function f is continuous in the closed bounded region R and holomorphic in the interior of R. Let

$$f(z) = u(z) + iv(z)$$

be the decomposition into real and imaginary parts. Suppose that f is not constant.

1. Show that neither u nor v are constant. (6 points)
2. Show that u attains its maximum value on the boundary of R. (6 points)
3. Show that v attains its maximum value on the boundary of R. (5 points)

(Hint: For the first part, use the Cauchy-Riemann equations. For the second part, consider $\exp(f(z)))$.

Problem 2: If

$$z_n := \frac{\cos(n)}{n^2} + i\frac{2 - n^2}{1 + 2n^2}$$

decide whether the sequence (z_n) converges. If it converges, find its limit. (16 points)

Problem 3: Decide whether the series $\sum_{n=0}^{\infty} e^{in}$ converges. If it converges, find its sum, i.e., its limit. (17 points)

Problem 4: For the function

$$f(z) = \frac{z + 5}{z^2 + z - 2}$$

find the Maclaurin series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

Make a table with the explicit values a_0, a_1, a_2, a_3, a_4, and a_5, and determine the radius of convergence of the Maclaurin series.
(Hint: Use partial fraction decomposition and the fact, which we have not yet shown in class, that a power series representation is unique, so that every power series representation must be the Maclaurin series. It is possible, but extremely

tedious, to use the explicit formulas for the coefficients of the Maclaurin series in this example. This remark also applies to the other problems on this sheet.)

(17 points)

Problem 5: For the function

$$f(z) = z^2 \cos(z)$$

find the Maclaurin series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

Make a table with the explicit values a_0, a_1, a_2, a_3, a_4, a_5, and a_{1001}, and determine the radius of convergence of the Maclaurin series. (16 points)

Problem 6: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find the Taylor series

$$f(z) = \sum_{n=0}^{\infty} a_n (z-1)^n$$

about the point 1. Make a table with the explicit values a_0, a_1, a_2, a_3, a_4, a_5, and a_{1001}, and determine the radius of convergence of the Taylor series.

(17 points)

Due date: Tuesday, November 17, 2015. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.