Memorial University of Newfoundland **Fall Semester 2015**
Yorck Sommerhäuser **MATH 3210: Sheet 5**

Introduction to Complex Analysis

Problem 1: For the function $f(z) = z^2$, compute the contour integrals

$$\int_{C_1} f(z)dz \qquad \text{and} \qquad \int_{C_2} f(z)dz$$

where C_1 is the curve parametrized by $\gamma_1(t) = t + it$ and C_2 is the curve parametrized by $\gamma_2(t) = t^2 + it^2$, where in both cases $0 \leq t \leq 1$. Draw a picture of both C_1 and C_2 in the Argand plane. Discuss in detail how the integrals are related: If they are equal, find a reason why they are equal; if they are different, explain how their difference comes about. (17 points)

Problem 2: For the function $f(z) = \frac{z+z^2}{z-2}$, compute the contour integrals

$$\int_{C_1} f(z)dz \qquad \text{and} \qquad \int_{C_2} f(z)dz$$

where C_1 is the curve parametrized by $\gamma_1(t) = 2 + 5e^{it}$ and C_2 is the curve parametrized by $\gamma_2(t) = 2 + 5e^{2it}$, where in both cases $0 \leq t \leq 2\pi$. Draw a picture of both C_1 and C_2 in the Argand plane. Discuss in detail how the integrals are related: If they are equal, find a reason why they are equal; if they are different, explain how their difference comes about. (17 points)

Problem 3: For the function $f(z) = \bar{z} - 1$, compute the contour integrals

$$\int_{C_1} f(z)dz \qquad \text{and} \qquad \int_{C_2} f(z)dz$$

where C_1 is the curve parametrized by $\gamma_1(t) = 1 + \cos(t) + i\sin(t)$ and C_2 is the curve parametrized by $\gamma_2(t) = 1 + \cos(t) - i\sin(t)$, where in both cases $0 \leq t \leq 2\pi$. Draw a picture of both C_1 and C_2 in the Argand plane. Discuss in detail how the integrals are related: If they are equal, find a reason why they are equal; if they are different, explain how their difference comes about. (17 points)

Problem 4: Let C be the following (piecewise smooth) contour consisting of a right semicircle of radius 4 connecting $2-3i$ to $2+5i$ followed by a line segment from $2+5i$ to $3+7i$. Give a parametrization of C. (There are many; try to find a particularly simple one.) Then compute the contour integral of

$$f(z) = y^2 - x + 3ix^2$$

where $z = x + iy$, over the contour C. (17 points)

Problem 5: Let C be any simple closed contour. Show that

$$\int_C z^5 dz = 0$$

(16 points)

Problem 6: Let $f(z) = u(z) + iv(z)$ be a function that is defined in some domain D, and let C be a contour in D. Find vector fields $F_1(x,y)$ and $F_2(x,y)$ so that the real part and the imaginary part of the contour integral

$$\int_C f(z)dz$$

are the line integrals of F_1 and F_2 in the sense of multivariable calculus, respectively, where as usual $z = x + iy$. (16 points)

Due date: Tuesday, October 27, 2015. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.