Memorial University of Newfoundland — **Fall Semester 2015**
Yorck Sommerhäuser — **MATH 3210: Sheet 4**

Introduction to Complex Analysis

Problem 1: With the help of complex differentiation rules, in particular the chain rule and the quotient rule, find the derivative of

$$f(z) = \left(\frac{z^3 + 8}{z^2 + 2i} \right)^5$$

(16 points)

Problem 2: For the function

$$f(z) := \begin{cases} \frac{\bar{z}^2}{z} & : z \neq 0 \\ 0 & : z = 0 \end{cases}$$

find all the points at which the Cauchy-Riemann equations are satisfied.
(Hint: Compare to Problem 6 on Sheet 3.) (17 points)

Problem 3: Use the Cauchy-Riemann equations in polar form (Sec. 24, Eq. (6), p. 69) to decide where the functions

1. $f(z) = \sqrt{r} e^{2\theta i}$ (for $r > 0$)
2. $g(z) = e^{-\theta} \cos(\ln(r)) + ie^{-\theta} \sin(\ln(r))$ (for $r > 0$)

are complex differentiable. If $f(z)$ or $g(z)$ are differentiable, find the derivative with the help of the formula for the derivative in polar coordinates (Sec. 24, p. 69). (8 points each)

Problem 4: Decide whether the function

$$h(x, y) = xy - x + y$$

is harmonic. If it is harmonic, find a harmonic conjugate. (17 points)

Problem 5: Let $f(z) = u(r,\theta) + iv(r,\theta)$, where $z = re^{i\theta}$, be a function that is infinitely often complex differentiable for all complex numbers z in a domain D. Show that

$$r^2 u_{rr}(r,\theta) + r u_r(r,\theta) + u_{\theta\theta}(r,\theta) = 0$$

and

$$r^2 v_{rr}(r,\theta) + r v_r(r,\theta) + v_{\theta\theta}(r,\theta) = 0$$

(Hint: Use the formulas mentioned in Problem 3.) (17 points)

Problem 6: Show that a real-valued analytic function, defined in a domain in the Argand plane, is constant. (17 points)

Due date: Tuesday, October 20, 2015. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.