Memorial University of Newfoundland **Fall Semester 2015**
Yorck Sommerhäuser **MATH 3210: Sheet 1**

Introduction to Complex Analysis

Problem 1: Show that multiplication of complex numbers is associative. In other words, show that for three complex numbers $z_1 = (a_1, b_1)$, $z_2 = (a_2, b_2)$, and $z_3 = (a_3, b_3)$, we have

$$((a_1, b_1)(a_2, b_2))(a_3, b_3) = (a_1, b_1)((a_2, b_2)(a_3, b_3))$$

or $(z_1 z_2)z_3 = z_1(z_2 z_3)$. (17 points)

Problem 2: Show that multiplication of complex numbers is distributive. In other words, using the notation of Problem 1, show that

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$$

(17 points)

Problem 3: Compute explicitly the complex number

$$\frac{3 - 2i}{4 - 3i} \cdot \frac{2 + i}{1 + 2i}$$

and find its real and imaginary parts. (16 points)

Problem 4: The equation $z^3 = 27i$ has three complex solutions, the three roots of $27i$. For each of these roots, find their real and imaginary part. In the Argand plane, i.e., the complex plane, draw a picture that shows these three roots. (17 points)

Problem 5: In the Argand plane, draw a picture of all complex numbers that satisfy the equations $|z - 2 + i| = 5$. Afterwards, do the same for the equation $|z - 2 + i| = |z + 2 - 3i|$. (16 points)

Problem 6: If $z = (\ln 2, \frac{7\pi}{6}) = \ln 2 + \frac{7\pi}{6} i$, compute explicitly the complex number e^z and find its real and imaginary parts. Simplify your answer so that no exponentials, logarithms, or trigonometric functions appear in the answer. (17 points)

Due date: Tuesday, September 22, 2015. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to copy down the problems again or to submit this sheet with your solution.