Scope of Inference

First, as this was an observational study, one cannot infer a causal relationship—that the longer humerus lengths among survivors enabled them to survive. Second, the living sparrows brought to Bumpus were found in a specific area and were so stressed that they were easily collected. Inference to populations of similarly stressed sparrows is risky. Such populations are hypothetical, and there is no chance model.

2.1.2 Anatomical Abnormalities Associated with Schizophrenia—An Observational Study

Are any physiological indicators associated with schizophrenia? Early studies, based largely on postmortem analysis, suggest that the sizes of certain areas of the brain may be different in persons afflicted with schizophrenia than in others. Confounding variables in these studies, however, clouded the issue considerably. In a 1990 article, researchers reported the results of a study that controlled for genetic and socioeconomic differences by examining 15 pairs of monozygotic twins, where one of the twins was schizophrenic and the other was not. The twins were located through an intensive search throughout Canada and the United States. (Data from R. L. Suddath et al., "Anatomical Abnormalities in the Brains of Monozygotic Twins Discordant for Schizophrenia," *New England Journal of Medicine* 322(12) (1990): 789–93.)

The researchers used magnetic resonance imaging to measure the volumes (in cm^3) of several regions and subregions inside the twins' brains. Display 2.2 presents data based on the reported summary statistics from one subregion, the left hippocampus. What is the magnitude of the difference in volumes of the left hippocampus between the unaffected and the affected individuals? Can the observed difference be attributed to chance?

Display 2.2 Differences in volumes (cm^3) of left hippocampus in 15 sets of monozygotic twins where one twin is affected by schizophrenia

Pair #	Unaffected	Affected	Difference
1	1.94	1.27	0.67
2	1.44	1.63	−0.19
3	1.56	1.47	0.09
4	1.58	1.39	0.19
5	2.06	1.93	0.13
6	1.66	1.26	0.40
7	1.75	1.71	0.04
8	1.77	1.67	0.10
9	1.78	1.28	0.50
10	1.92	1.85	0.07
11	1.25	1.02	0.23
12	1.93	1.34	0.59
13	2.04	2.02	0.02
14	1.62	1.59	0.03
15	2.08	1.97	0.11

Differences

−2	
−1	9
−0	
0	23479
1	0139
2	3
3	
4	0
5	09
6	7
7	

Legend: 6 | 7 represents 0.67 cm^3

Average: 0.199
Sample SD: 0.238
n: 15

Summary of Statistical Analysis

There is substantial evidence that the mean difference in left hippocampus volumes between schizophrenic individuals and their nonschizophrenic twins is nonzero (two-sided p-value = .006, from a paired t-test). It is estimated that the mean volume is 0.20 cm^3 smaller for those with schizophrenia (about 11% smaller). A 95% confidence interval for the difference is from 0.07 to 0.33 cm^3.

Scope of Inference

These twins were not randomly selected from general populations of schizophrenic and nonschizophrenic individuals. Tempting as it is to draw inferences to these wider populations, such inferences must be based on an assumption that these individuals are as representative as random samples are. Furthermore, the study is observational, so no causal connection between left hippocampus volume and schizophrenia can be established from the statistics alone. In fact, the researchers had no theories about whether the abnormalities preceded the disease or resulted from it.

2.2 One-Sample *t*-Tools and the Paired *t*-Test

The schizophrenia study used a *paired* t-test, in which measurements taken on paired subjects are reduced to a single set of differences for analysis. This section develops the single population methods for drawing inferences about the population mean from a random sample and at the same time introduces key concepts such as sampling distributions, standard errors, Z-ratios, and t-ratios.

2.2.1 The Sampling Distribution of a Sample Average

A random sample is drawn from a population with the objective of learning about the population's mean. Suppose the average of that sample is written on a piece of paper, which is placed in a box. Then suppose this process is repeated for every one of the equally likely samples that could be drawn. Then the distribution of all the numbers in the box is the *sampling distribution of the average*.

Display 2.3 illustrates a sampling distribution in the conceptual framework of the schizophrenia study. There is an assumed population of twins in which one of the twins has schizophrenia and the other does not. For each set of twins, Y represents the difference between the left hippocampus volumes of the unaffected and the affected twin. The 15 observed differences are assumed to be a random sample from this population. To examine whether there is a structural difference between volumes, one calculates the average of the 15 measurements, $\overline{Y} = 0.20\,cm^3$ as an estimate of the population mean μ.

Although only the one sample is actually taken, it is important to think about replicating the study—repeatedly collecting 15 sets of twins and repeatedly calculating the average difference. The value of the average varies from sample to sample, and a histogram of its values represents its sampling distribution.