Statistics 3540
Assignment #5: March 12, 2008
Due in class: Friday, March 28, 2008

If it appears that you can do a problem either by hand or using a statistical software package, the method you choose to use is up to you.

1. The Holt-Winters Method for additive seasonal models was used to analyze a time series collected every 3 months for 5 years. Some of the results are below, using $\alpha = \gamma = \delta = .03$:

$$l_{11} = 140.133,\ l_{12} = 144.872,\ b_{11} = 4.712,\ b_{12} = 4.714$$

$$sn_9 = -26.00,\ sn_{10} = 9.06,\ sn_{11} = 47.79,\ sn_{12} = -0.66$$

Suppose that $y_{13} = 115$. Find:
(a) l_{13} (b) b_{13} (c) sn_{13} (d) $\hat{y}_{13}(12)$

2. For the unemployment data that you have used in previous assignments:
   (a) Use the Holt-Winters Method for multiplicative seasonal models to analyze the data, using the following choices of smoothing constants:
   (i) $\alpha = \gamma = \delta = 0.1$
   (ii) $\alpha = 0.03, \gamma = 0.02, \delta = 0.05$
   (iii) $\alpha = \gamma = \delta = 0.5$
   (iv) $\alpha = \gamma = \delta = 0.8$
   (v) $\alpha = \gamma = 0.1,\ \delta = 0.9$.
   (b) Based on (a), what choices will you make for the smoothing constants? Explain.
   (c) Based on your choice in (b), find a 95% prediction intervals for the unemployment rate in January and February 2008. **You may use the intervals that Minitab gives**, *i.e.* you do not have to calculate the intervals by hand.

3. For the unemployment data discussed in the previous question:
   (a) Analyze these data using the Holt-Winters Method for additive models. Use the smoothing constants shown in #2.
   (b) Based on (a), what choices will you make for the smoothing constants? Explain.
   (c) Based on your choice in (b), find 95% prediction intervals for fruit imports in January and February 2008. **Find these intervals by hand**, *i.e.* do not use the intervals that Minitab gives.
   (d) Does it appear that the additive method is more appropriate for analyzing these data? Explain.

4. For the time series model

$$y_t = (5a_t - 2a_{t-1})/3$$

where a_t are independent $\mathrm{N}(0, \sigma)$, find and plot the autocorrelation function $\rho(k)$.

5. Consider the time series model

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

where ϵ_t are independent $\mathrm{N}(0, \sigma)$.

   (a) Show that this model is nonstationary.

   (b) Define a new time series model $z_t = y_t - y_{t-1}$. Show that this transformed series has a mean and variance that do not depend on t.

6. For the AR(3) model

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + a_t, \qquad t = 1, \ldots, n$$

where a_t are independent, $a_t \sim \mathrm{N}(0, \sigma)$, it can be shown that

$$\phi_2 R(0) = R(2) - (\phi_1 + \phi_3) R(1)$$

   (a) Find an expression for $\rho(2)$ for the AR(3) model.

   (b) Would it be possible to have $\phi_2 = \phi_3 = -1.1$ and $\phi_1 = 1.1$ in an AR(3) model? Explain why or why not.

7. Data on the yields from 70 consecutive runs (1 run per day) of a chemical process have been collected. The data is available on the course website in the file **chem.mtw**.

   (a) Plot the data. Does the series appear to have (i) constant mean (ii) constant variability?

   (b) Plot the SAC and SPAC. Does the series appear stationary?

   (c) Based on your plots, suggest a time series model that could be fit to this data.

   (d) Fit an AR(1) model

$$y_t = \delta + \phi y_{t-1} + a_t$$

   to the data, where a_t are independent $\mathrm{N}(0, \sigma)$.