Statistics 3540
Assignment #1: Jan. 11, 2008
Due in class: Jan. 21, 2008

1. This problem will get you to use the world of www to perform a small simulation study to get a feel for the concept of the sampling distribution of $\bar{x}$ and the central limit theorem. What you will do is run a program that will draw repeated samples from a normal distribution and observe the behaviour of $\bar{x}$. This will be a multi-part exercise.

   Using your favourite web browser, go to the following site:

   `http://onlinestatbook.com/rvls.html`

   Once at this site, click on

   Simulations/Demonstrations – Sampling Distribution Simulation

   then the **Begin** box on the left–hand side of the screen.

   (a) A histogram appears at the top of the page, depicting a normal distribution. What are you told the mean (μ) and standard deviation (σ, written as sd on the screen) are?

   (b) You are going to get the computer to draw samples from the distribution shown in (a), and calculate the mean of each of the samples. You will first do this by drawing 5 samples from this distribution. Each sample will contain $n = 20$ observations. **NOTE: The computer screen calls this** N.

   To do this, go to the third plot area on the screen, entitled **Distribution of Means**. Change the value of N to $N = 20$.

   Then, go by the second plotting area and click the box **5**. This will choose 5 samples, each with 20 values, and find the mean of each sample ($\bar{x}$).

   (c) A histogram of these $\bar{x}$ should appear. Describe its shape.

   (d) What are the mean and and standard deviation of the $\bar{x}$ values? How do they compare with what the Central Limit Theorem says the theoretical mean (μ) and standard deviation ($\sigma/\sqrt{n}$) of the sampling distribution will be?

   (e) Next, click on the box that says **1,000**. This will draw 1000 samples from your normal distribution, and calculate the corresponding 1000 $\bar{x}$ values.

   (f) Describe the shape of the new histogram that is produced.

(g) What are the mean and standard deviation of these $\bar{x}$ values? Are they "closer" to what the Central Limit Theorem says should happen?

2. We will now do a very small simulation to illustrate the interpretation of a confidence interval.

   (a) In Minitab, you first want to create 60 samples of size $n = 50$ from a normal distribution with mean = 10 and standard deviation (σ) of 3. This can be done as follows:

   (1) Click on `Calc-Random Data-Normal`, and specify you want 50 rows and 60 columns. Make sure to use the correct mean and standard deviation for this example. Then hit `OK`. The 60 samples are in the columns C1–C60.

   (2) To find the confidence interval for μ for each sample:

   Click on `Stat-Basic Statistics-1-Sample Z`

   The samples are columns C1-C60. Be sure to specify the correct standard deviation, and hit `OK`. The 60 CI's will now appear in your `Session` window.

   (b) How many of your 60 intervals contain the true μ?

   (c) According to the theoretical result, how many of the CI's would you expect to contain the true μ? (See p. 53 in the textbook, or p. 52 of the text on reserve in the library).

3. Refer to problem #2.16, p. 77. Change the value $s = 6.02$ feet to $\sigma = 6.02$ feet. Now answer the following:

   (a) Calculate a 99% confidence interval for the mean stopping distance.

   (b) Problem #2.19, p. 77.

4. Refer to Problem #3.1, p. 125 and complete the following. Do all parts of this question by hand.

   (a) 3.1 (a, b).

   (b) Calculate the least squares estimates of β_0 and β_1.

   (c) Calculate the residual for a student with a GPA of 2.60.

5. Problem #3.5, p. 125–126. Also find $\hat{\beta}_1$, R^2, r and $\hat{\sigma}^2$. Do this problem in Minitab or another software package (SPSS, SAS, etc.).