Estimation in ARMA and ARIMA Models

EX 1: Consider the 253 daily returns for 1986 of the S & P equally weighted index, studied previously. Our SAC and SPAC plots indicated that either an AR(1), MA(1) or ARMA(1,1) model may be appropriate. We will use Minitab to fit these models.

To fit the models using Minitab, choose

Stat – Time Series – ARIMA

and supply the number of AR terms, MA terms and number if differences (if fitting an ARIMA model).

The output for the AR(1) model is below (so 1 AR term, 0 MA terms, 0 differences):

ARIMA Model: SP500

```
Estimates at each iteration

Iteration      SSE   Parameters
        0  2.06063  0.100  0.091
        1  0.05605  0.209  0.014
        2  0.00982  0.237  0.002
        3  0.00951  0.234  0.001
        4  0.00951  0.233  0.001
        5  0.00951  0.233  0.001

Relative change in each estimate less than 0.0010

Final Estimates of Parameters

Type           Coef    SE Coef     T      P
AR   1       0.2329     0.0615  3.79  0.000
Constant  0.0005336  0.0003869  1.38  0.169
Mean      0.0006957  0.0005044

Number of observations:  253
Residuals:    SS =  0.00950710 (backforecasts excluded)
              MS =  0.00003788  DF = 251

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag             12     24     36     48
Chi-Square     6.1   14.4   20.2   35.5
DF              10     22     34     46
P-Value     0.808  0.887  0.971  0.870
```

ARMA(1,1)

```
Final Estimates of Parameters

Type           Coef    SE Coef      T      P
AR   1      -0.0609     0.2476  -0.25  0.806
MA   1      -0.3114     0.2357  -1.32  0.188
Constant  0.0007335  0.0005073   1.45  0.149
Mean      0.0006914  0.0004781


Number of observations:  253
Residuals:    SS =  0.00945779 (backforecasts excluded)
              MS =  0.00003783  DF = 250
```

MA(1), with forecasts

```
Final Estimates of Parameters

Type           Coef    SE Coef      T      P
MA   1      -0.2543     0.0612  -4.16  0.000
Constant  0.0006925  0.0004842   1.43  0.154
Mean      0.0006925  0.0004842


Number of observations:  253
Residuals:    SS =  0.00945987 (backforecasts excluded)
              MS =  0.00003769  DF = 251

Forecasts from period 253

                         95% Limits
Period   Forecast       Lower      Upper  Actual
   254  0.0021918  -0.0098433  0.0142269
   255  0.0006925  -0.0117255  0.0131106
   256  0.0006925  -0.0117255  0.0131106
```

EX 2: University of Iowa annual enrollment. The SAC and SPAC plots suggest the first differences are stationary. We will fit the 4 models suggested in class.

ARIMA(2, 1, 1) model, with forecasts. Note that we have some convergence problems with this model. There are still large changes happening with the parameter estimates, but the SSE can't be reduced any further. This suggests some problem with this model.

```
ARIMA Model: Enrol

Estimates at each iteration

Iteration       SSE            Parameters
        0  26073467  0.100   0.100   0.100  892.194
        1  21155353  0.159  -0.007  -0.050  947.153
        2  20652904  0.309  -0.043   0.076  819.356
        3  19823533  0.459  -0.089   0.185  703.355
        4  17432725  0.609  -0.183   0.188  641.113
        5  15590388  0.759  -0.288   0.165  589.723
        6  14781577  0.909  -0.384   0.169  527.711
        7  14644388  1.001  -0.443   0.189  488.242
        8  14638792  1.027  -0.460   0.204  477.002

Unable to reduce sum of squares any further

Final Estimates of Parameters

Type         Coef  SE Coef      T      P
AR   1     1.0269   0.4979   2.06  0.050
AR   2    -0.4598   0.3265  -1.41  0.172
MA   1     0.2036   0.5537   0.37  0.716
Constant   477.0    115.8   4.12  0.000

Differencing: 1 regular difference
Number of observations:  Original series 29, after differencing 28
Residuals:    SS =  14176524 (backforecasts excluded)
              MS =  590688  DF = 24

Forecasts from period 29

                     95% Limits
Period  Forecast    Lower    Upper  Actual
    30   46854.6  45347.9  48361.3
    31   48179.3  45046.1  51312.4
    32   49426.8  44856.0  53997.6
```

ARIMA (1, 1, 0) Model, with forecasts

```
Final Estimates of Parameters

Type        Coef  SE Coef     T      P
AR   1    0.6340   0.1514  4.19  0.000
Constant   383.0    149.4  2.56  0.017


Differencing: 1 regular difference
Number of observations:  Original series 29, after differencing 28
Residuals:    SS =  16226166 (backforecasts excluded)
              MS =  624083  DF = 26


Forecasts from period 29

                    95% Limits
Period  Forecast    Lower    Upper  Actual
    30   46602.3  45053.7  48151.0
    31   47638.6  44671.8  50605.5
    32   48678.6  44349.2  53008.1
```

ARIMA (2, 1, 0) Model, with forecasts

```
Final Estimates of Parameters

Type         Coef  SE Coef      T      P
AR   1     0.8475   0.1867   4.54  0.000
AR   2    -0.3451   0.1880  -1.84  0.078
Constant    544.3    142.8   3.81  0.001


Differencing: 1 regular difference
Number of observations:  Original series 29, after differencing 28
Residuals:    SS =  14271133 (backforecasts excluded)
              MS =  570845  DF = 25

Forecasts from period 29

                    95% Limits
Period  Forecast    Lower    Upper  Actual
    30   46845.7  45364.5  48326.9
    31   48117.2  45005.6  51228.8
    32   49299.7  44771.8  53827.6
```

ARIMA(0, 1, 1) Model, with forecasts

```
Final Estimates of Parameters

Type         Coef  SE Coef      T      P
MA   1    -0.5854   0.1693  -3.46  0.002
Constant   1103.8    240.5   4.59  0.000


Differencing: 1 regular difference
Number of observations:  Original series 29, after differencing 28
Residuals:    SS =  16916880 (backforecasts excluded)
              MS =  650649  DF = 26

Forecasts from period 29

                     95% Limits
Period  Forecast    Lower    Upper  Actual
    30   46958.0  45376.7  48539.3
    31   48061.8  45097.8  51025.8
    32   49165.7  45283.6  53047.7
```

To illustrate that the estimation can be done in two steps, here are the results when the first differences are found separately, then an MA(1) model is fit to the first differences. We see the results match the ARIMA(0, 1, 1) results, except something is wrong with the forecasts. What is the problem?

```
Final Estimates of Parameters

Type         Coef  SE Coef      T      P
MA   1    -0.5854   0.1693  -3.46  0.002
Constant   1103.8    240.5   4.59  0.000
Mean       1103.8    240.5


Number of observations:  28
Residuals:    SS =  16916880 (backforecasts excluded)
              MS =  650649  DF = 26

Forecasts from period 29

                     95% Limits
Period  Forecast    Lower    Upper  Actual
    30   1386.03  -195.28  2967.34
    31   1103.81  -728.50  2936.12
    32   1103.81  -728.50  2936.12
```